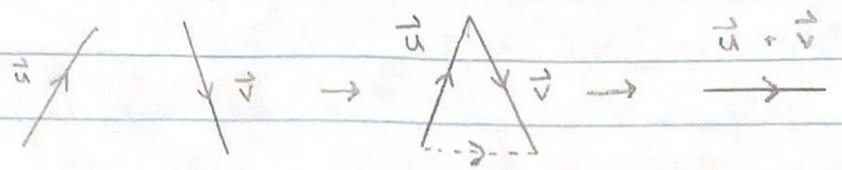
Friday, August 27th Calc III Notes

Operations on Vectors:

① Magnitude (Vector $\rightarrow$ Real # ≥ 0)

"length of a segment

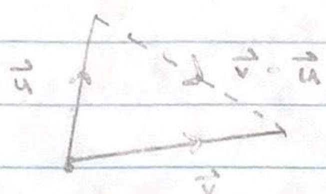
② Addition (vector + vector $\rightarrow$ Vector)



"head to tail", Addition is done via the "parallelogram law"

③ Subtraction (Vector - Vector $\rightarrow$ Vector)

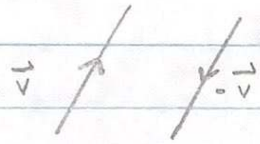
"tip to tip"



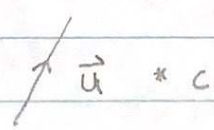
NOTE: Arrow points to first vector

④ Negation (Vector $\rightarrow$ Vector)

$-\vec{v}$ is obtained from $\vec{v}$ by "flipping it"



⑤ Scalar Multiplication (Scalar * Vector $\rightarrow$ Vector)

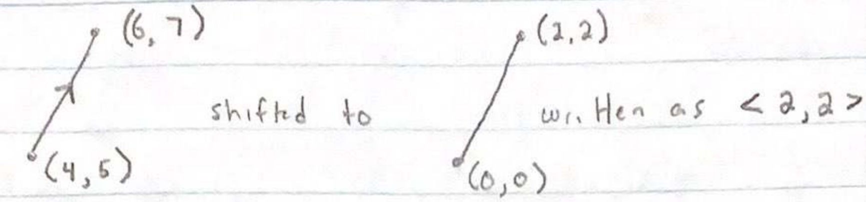


if $c > 1$, vector gets stretched

if $0 < c < 1$, vector gets squished

if $c < 0$, vector gets flipped

Every vector has a unique representation with tail at origin



x component $(6-4) = 2$, y component $(7-5) = 2$
(zero vector has components of 0)

Vector Operations rewritten with components
(Let $\vec{u}$ be $\langle u_1, u_2, u_3 \rangle$, $\vec{v}$ be $\langle v_1, v_2, v_3 \rangle$)

1. Magnitude: $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$
   derived from distance formula
2. Addition: $\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$
3. Subtraction: $\vec{u} - \vec{v} = \langle u_1 - v_1, u_2 - v_2, u_3 - v_3 \rangle$
4. Negation: $-\vec{u} = \langle -u_1, -u_2, -u_3 \rangle$
5. Scalar Multiplication: $c * \vec{u} = \langle cu_1, cu_2, cu_3 \rangle$

Theorem (Properties of Vector Operations)

1. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ Associative Property
2. $\vec{u} + \vec{v} + \vec{w} = \vec{v} + \vec{w} + \vec{u}$ Commutative Property
3. $\vec{0} + \vec{u} = \vec{u}$ Identity Property
4. $\vec{v} - \vec{v} = \vec{0}$
5. $a(b\vec{v}) = (ab)\vec{v}$
6. $(a+b)\vec{v} = a\vec{v} + b\vec{v}$
7. $a(\vec{v} + \vec{u}) = a\vec{v} + b\vec{v}$
8. $0\vec{v} = \vec{0}$, $1\vec{v} = \vec{v}$

Note: Have to be bound to same plane ($\mathbb{R}^2$, $\mathbb{R}^3$, etc)

$\langle -1, 2 \rangle + \langle 0, 1, 2 \rangle$ is nonsense

Note: Scalar multiplication NEEDS scalar quantity

Direction:

Given a vector $\vec{u}$

Standard basis in $\mathbb{R}^3$

$i = \langle 1, 0, 0 \rangle$

$j = \langle 0, 1, 0 \rangle$

$k = \langle 0, 0, 1 \rangle$

Every vector $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$= \langle v_1, 0, 0 \rangle + \langle 0, v_2, 0 \rangle + \langle 0, 0, v_3 \rangle$